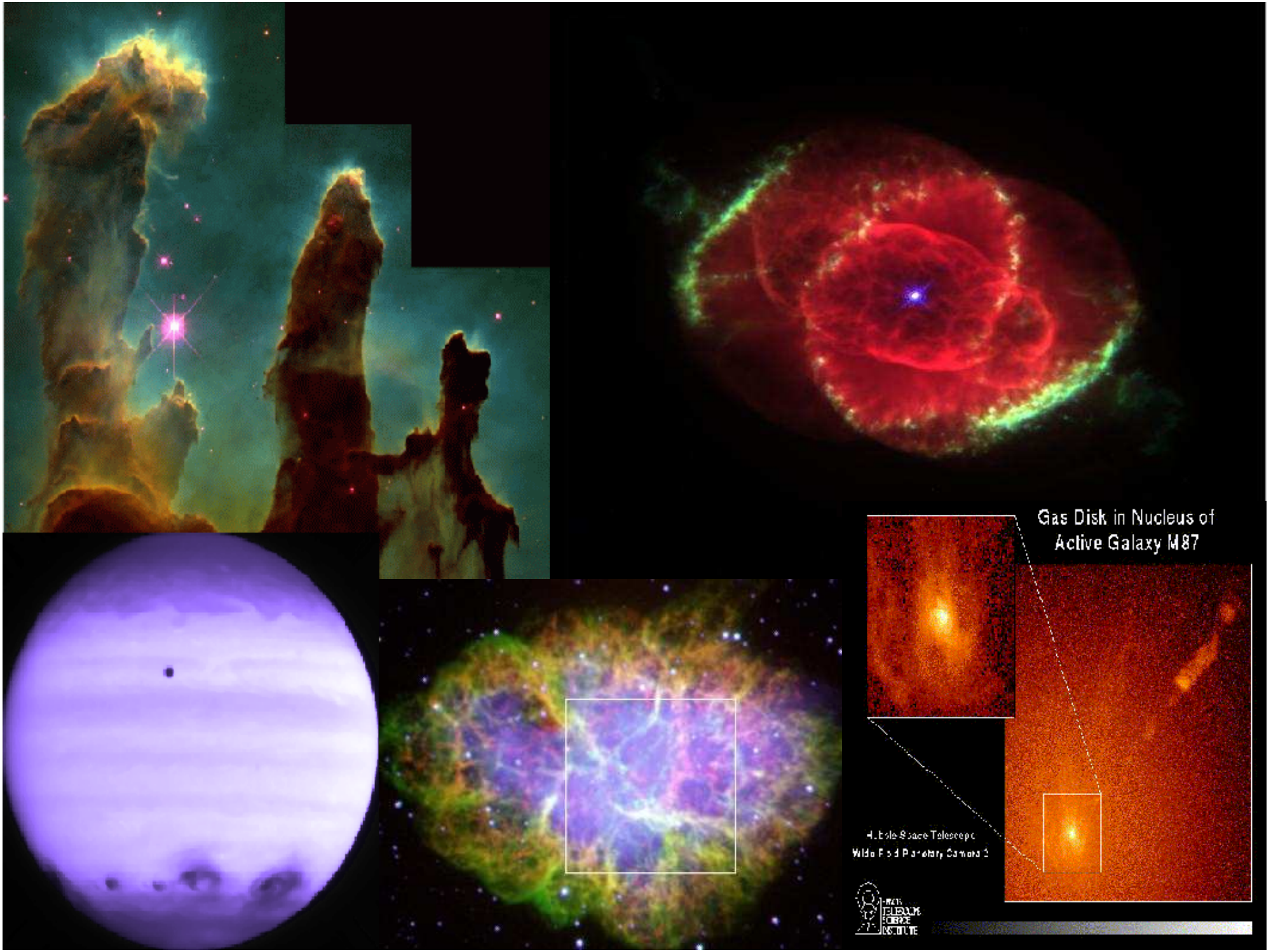
A- MAZE:
A first glance on the concepts

codes
visualization
simulation control


Gas Disk in Nucleus of
Active Galaxy M87

Modeling results in a complex system of equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{v})=0$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}[\rho \boldsymbol{v}\cdot\boldsymbol{v}+p\boldsymbol{I}-\theta_{\nu_v}]=\boldsymbol{f}(\boldsymbol{x},t)$$

$$\frac{\partial E}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}[\boldsymbol{v}(E+p)-\theta_{\nu_v}\cdot\boldsymbol{v}-\kappa_C T^{5/2}\frac{\partial T}{\partial \boldsymbol{x}}]=\boldsymbol{v}\cdot\boldsymbol{f}-C+H$$

$$\frac{1}{c}\frac{\partial I^{\nu_f}}{\partial t}+(\underline{n}\cdot\nabla)I^{\nu_f}=\eta^{\nu_f}-(\kappa^{\nu_f}+\lambda^{\nu_f})I^{\nu_f}+\lambda^{\nu_f}\oint_{S_2} d\Omega I^{\nu_f}$$

$$\partial Z_i/\partial t=R(Z^i,I,\boldsymbol{x},t) \qquad \Rightarrow f_R, C_R, H_R$$

$$\Delta\Phi=\rho \qquad \Rightarrow f_G=\nabla\Phi$$

Types of differential equations

Historically: $a\,q_{xx} + b\,q_{xy} + c\,q_{yy} + d\,q_x + e\,q_y + f\,q = g$

$$D \equiv b^2 - 4ac$$

Elliptic : D < 0, complex eigenvalues

parabolic : D = 0, complex eigenvalues, one eigenvalue =∞

hyperbolic: D > 0, real eigenvalues

M odel equations

$\Delta \Phi = \rho$ **Poisson equation**

$\frac{\partial T}{\partial t} - \chi_C \Delta T = 0$ **diffusion equation (heat conduction)**

$q_{tt} = q_{xx}$ **Wave equation**

Elliptic equations

$\Delta \Phi = \rho$ **Poisson equation**

Parabolic equations

$$\frac{\partial T}{\partial t} - \chi_C \Delta T = 0$$

diffusion equation (heat conduction)

$$T(x,0) = Q_0 \delta(x)$$

$$T(x,t) = \frac{Q_0}{\sqrt{4\pi\chi_C t}} \exp\left(\frac{-x^2}{4\chi_C t}\right)$$

Central differences, finite elements, spectral methods

Stiffness:

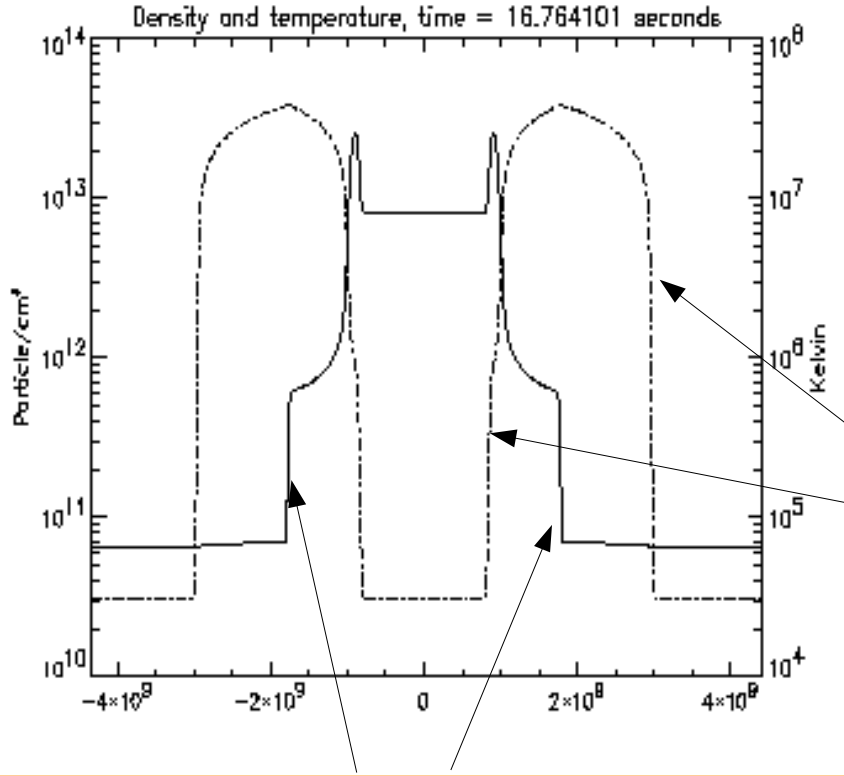
sinosoidal disturbunces: sin(ξx) will decay as exp(- ξ^2 t)

----> any explicit numerical method has a stability criterion $\Delta t \sim \Delta x^2$

Non- linear equation describes heat conduction in a plasma
- by thermal electrons and ions
- by photon- diffusion

$$\frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \kappa_C T^{\beta} \frac{\partial T}{\partial x} = 0$$

β = 3/2 in weekly ionized media
β = 5/2 in well ionized plasmae
β = 7/2 for photon- diffusion



heat- fronts:
non- classical solution possible

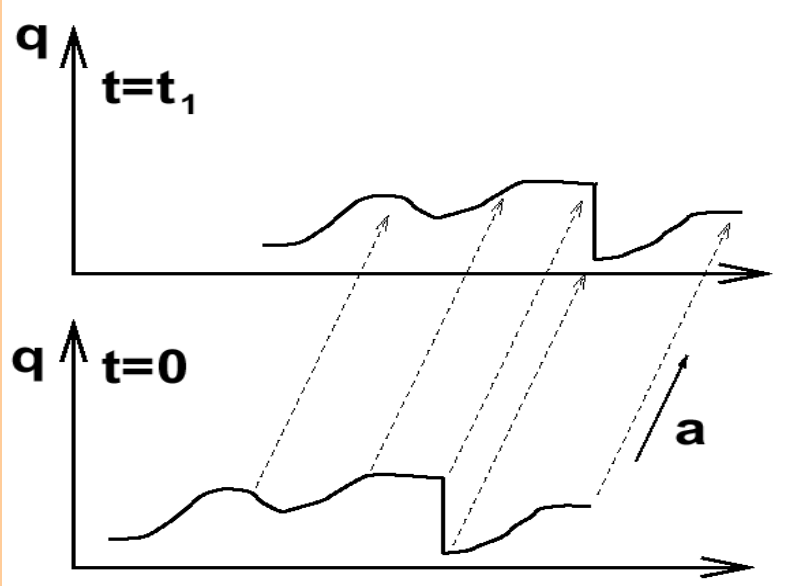
shocks

Hyperbolic equations

$$q_t + a \; q_x = 0$$

Wave equation

$$q(x,t) = q(x - at, 0)$$



$$q_t \; + f(q)_x = 0$$

Non- classical solutions:
shocks, tangential discontinuities

Finite Volume methods, discontinuous finite elements,
spectral methods (linear case)

Radiative Transfer Equation

$$\frac{1}{c}\frac{\partial I^{\nu_f}}{\partial t} + (\underline{n} \cdot \nabla) I^{\nu_f} = \eta^{\nu_f} - (\kappa^{\nu_f} + \lambda^{\nu_f}) I^{\nu_f} + \lambda^{\nu_f} \oint_{S_2} d\Omega\, I^{\nu_f}$$

- 'Infinitely many' equations,
  integral- term couples all the directions
  ---> need discretization which accounts for this

- different regimes
  - transport (hyperbolic) regime, if κ, λ ----> 0
  - diffusive (parabolic) regime, if scattering dominates

Modeling results in a complex system of equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{v}) = 0$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}[\rho \boldsymbol{v} \cdot \boldsymbol{v} + p\boldsymbol{I} - \theta_{\nu_v}] = \boldsymbol{f}(\boldsymbol{x}, t)$$

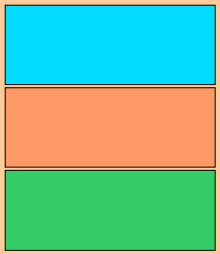
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}[\boldsymbol{v}(E+p) - \theta_{\nu_v} \cdot \boldsymbol{v} - \kappa_C T^{5/2} \frac{\partial T}{\partial \boldsymbol{x}}] = \boldsymbol{v} \cdot \boldsymbol{f} - C + H$$

$$\frac{1}{c}\frac{\partial I^{\nu_f}}{\partial t} + (\underline{n} \cdot \nabla) I^{\nu_f} = \eta^{\nu_f} - (\kappa^{\nu_f} + \lambda^{\nu_f}) I^{\nu_f} + \lambda^{\nu_f} \oint_{S_2} d\Omega I^{\nu_f}$$

$$\frac{\partial Z_i}{\partial t} = R(Z^i, I, \boldsymbol{x}, t) \qquad \Rightarrow f_R, C_R, H_R$$

$$\Delta \Phi = \rho \qquad \Rightarrow f_G = \nabla \Phi$$

fractional step methods:

Instead of solving  for one time-step

Solve , and for one time-step

with appropriate initial conditions and in appropriate order

Advantages

- ✗ **compute the part-problems with well established solvers**
- ✗ **chose problem adapted solver for each part-problem**
- ✗ **test the solution of each part-problem**
- ✗ **understand physically each part-problem**

An example system

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{v}) = 0$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{v} \cdot \boldsymbol{v} + p) = \boldsymbol{f}(\boldsymbol{x}, t)$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}\left[\boldsymbol{v}(E + p)\right] - \frac{\partial}{\partial \boldsymbol{x}}(\kappa T^{5/2} \frac{\partial T}{\partial \boldsymbol{x}}) = \boldsymbol{v} \cdot \boldsymbol{f} - C$$

$$(\underline{\boldsymbol{n}} \cdot \nabla) I^{\nu} = \eta^{\nu} - (\kappa^{\nu} + \lambda^{\nu}) I^{\nu} + \lambda^{\nu} \oint_{S_2} d\Omega \, I^{\nu}$$

fractional steps for evolving the system for one timestep: $t_n \longmapsto t_{n+1} = t_n + \Delta t$

solve homogeneous, modified Euler equations with initial data $\underline{u}^n = (\rho^n, \Pi_x^n, \Pi_y^n, \Pi_z^n, E^n)^t$ for time-step $\Delta t/2$ to obtain $\underline{\tilde{u}}^{n+1/2}$

solve stationary radiative transfer on $\underline{\tilde{u}}^{n+1/2}$ to obtain $\tilde{I}_\nu = I_\nu(x, t^{n+1/2}, \underline{n})$

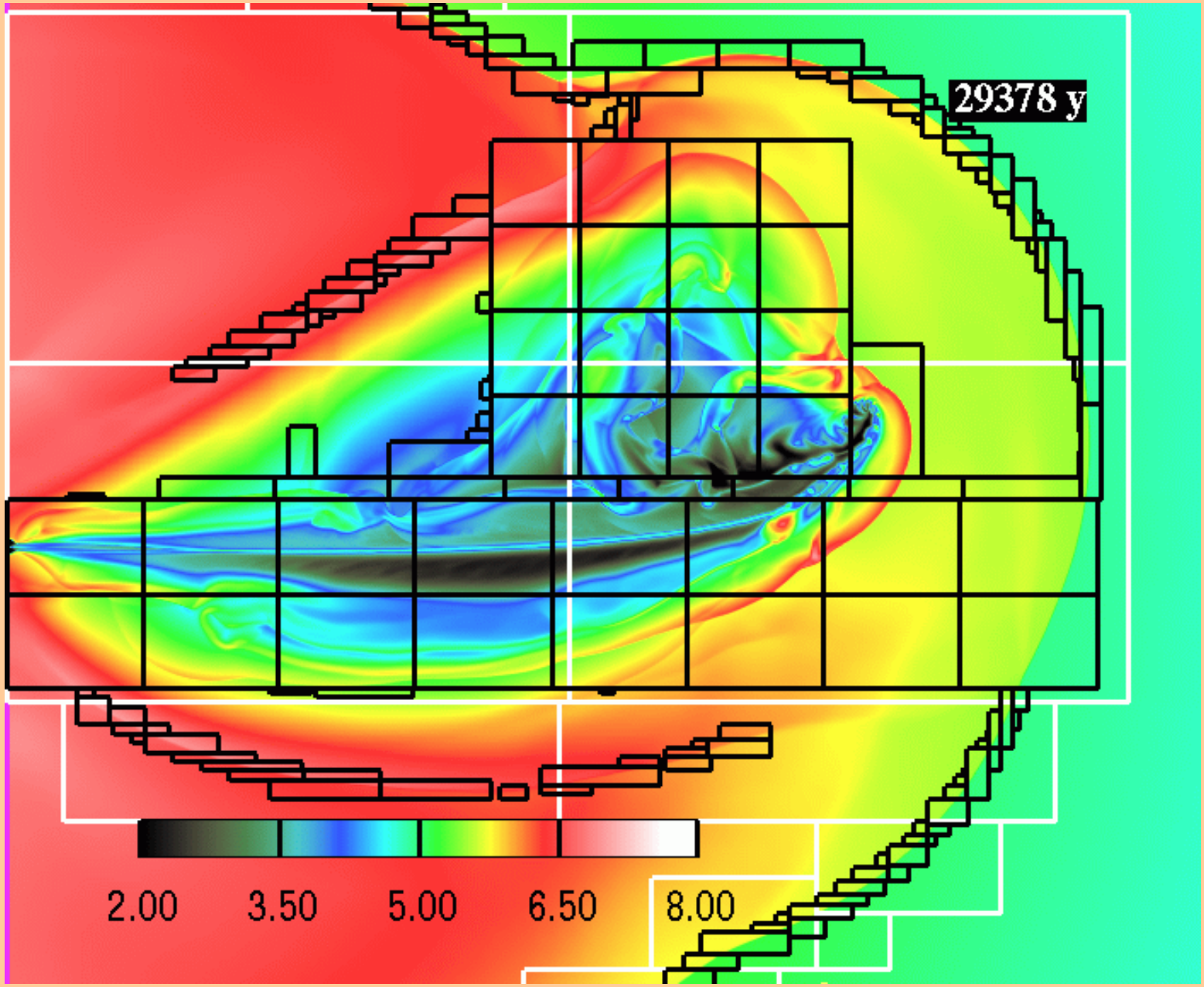
compute gravity, radiative forces, cooling on $(\underline{\tilde{u}}^{n+1/2}, I_\nu)$

solve ODE for the sources, $\partial u/\partial t = S$, with initial data $\underline{\tilde{u}}^{n+1/2}$ for time-step Δt to obain $\underline{\tilde{\tilde{u}}}^{n+1/2}$

solve homogeneous, modified Euler equations with initial data $\underline{\tilde{\tilde{u}}}^{n+1/2}$ for time-step $\Delta t/2$ to obtain $\underline{u}^{n+1}$

AMRCART

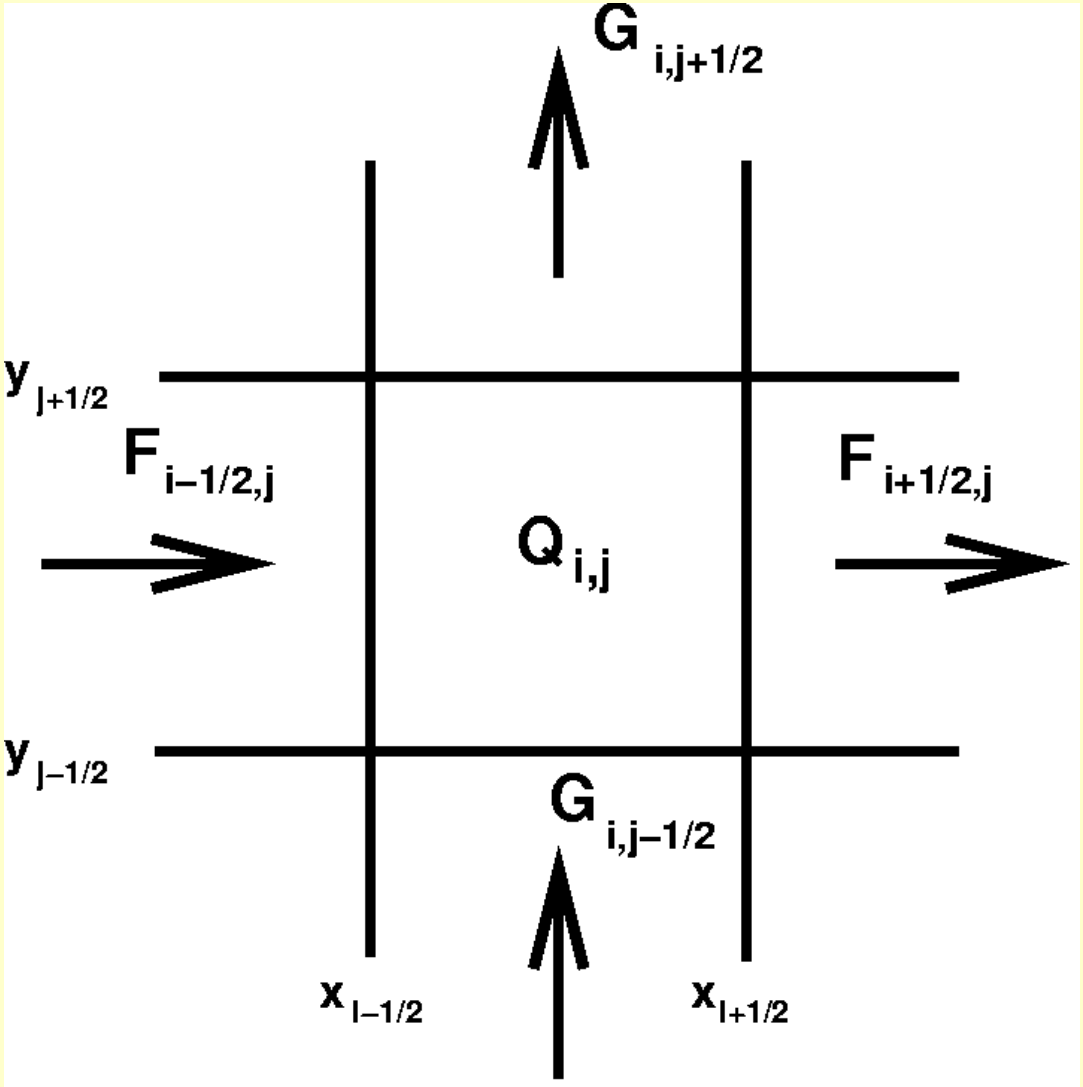
1D - 3D HYDRO and MHD


29378 y
2.00
3.50
5.00
6.50
8.00

Needs:

- Integration algorithm
- Adptive mesh algorithm
- Visualization
- Data management

Finite Volume Methods



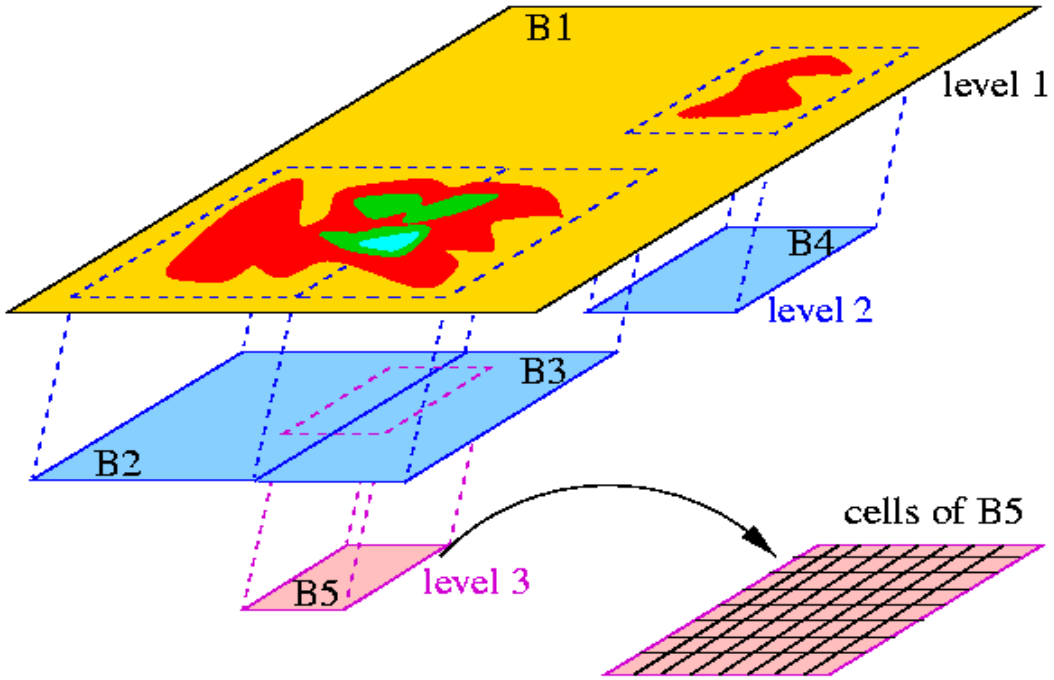
conservative

upwinding

TVD

high order of integration ----> high accuracy

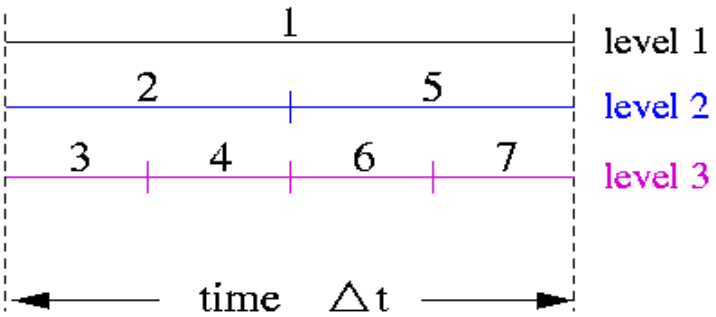
block structured A M R



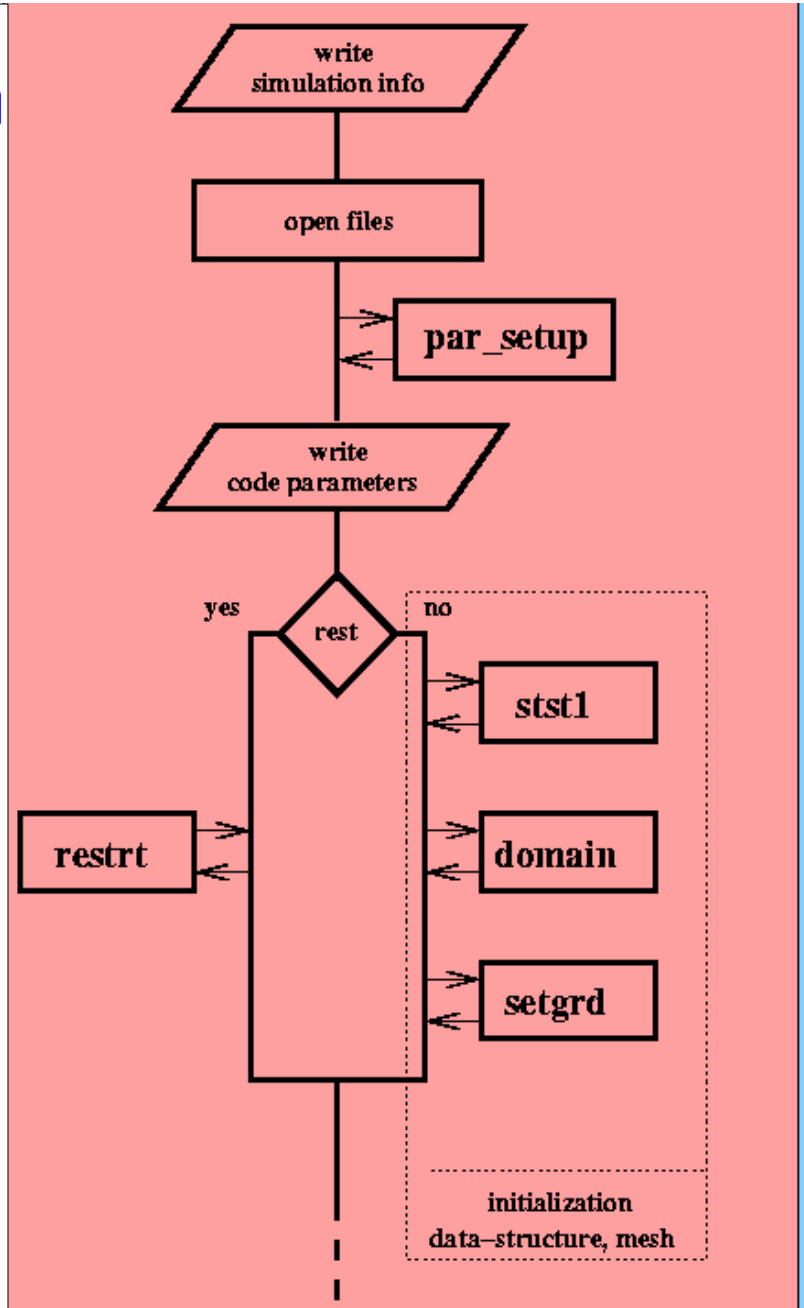
storing the grid: list of independent blocks
refinement ratios between levels

storing a block: l_x, l_y, l_z, one corner of each block
pointer to ...

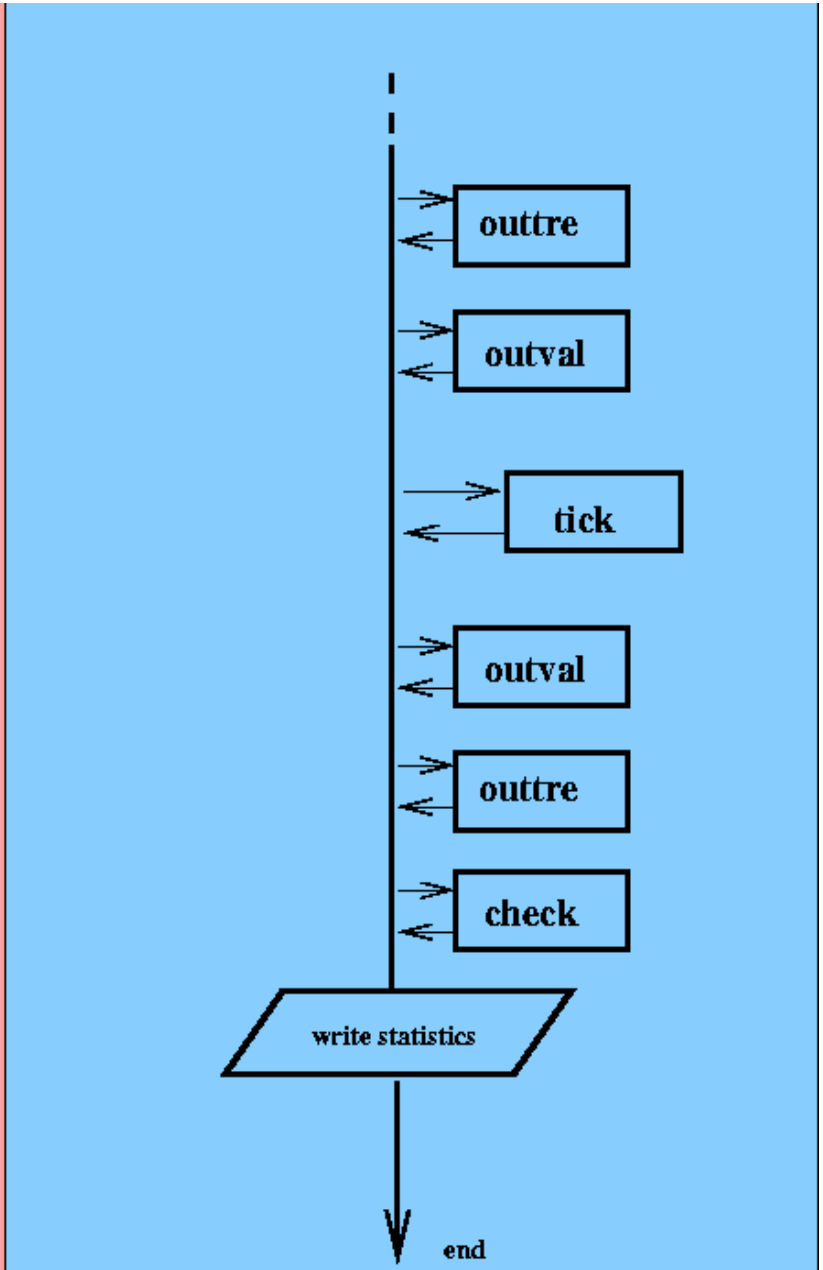
integration scheme:



main program



initialization



call for driver

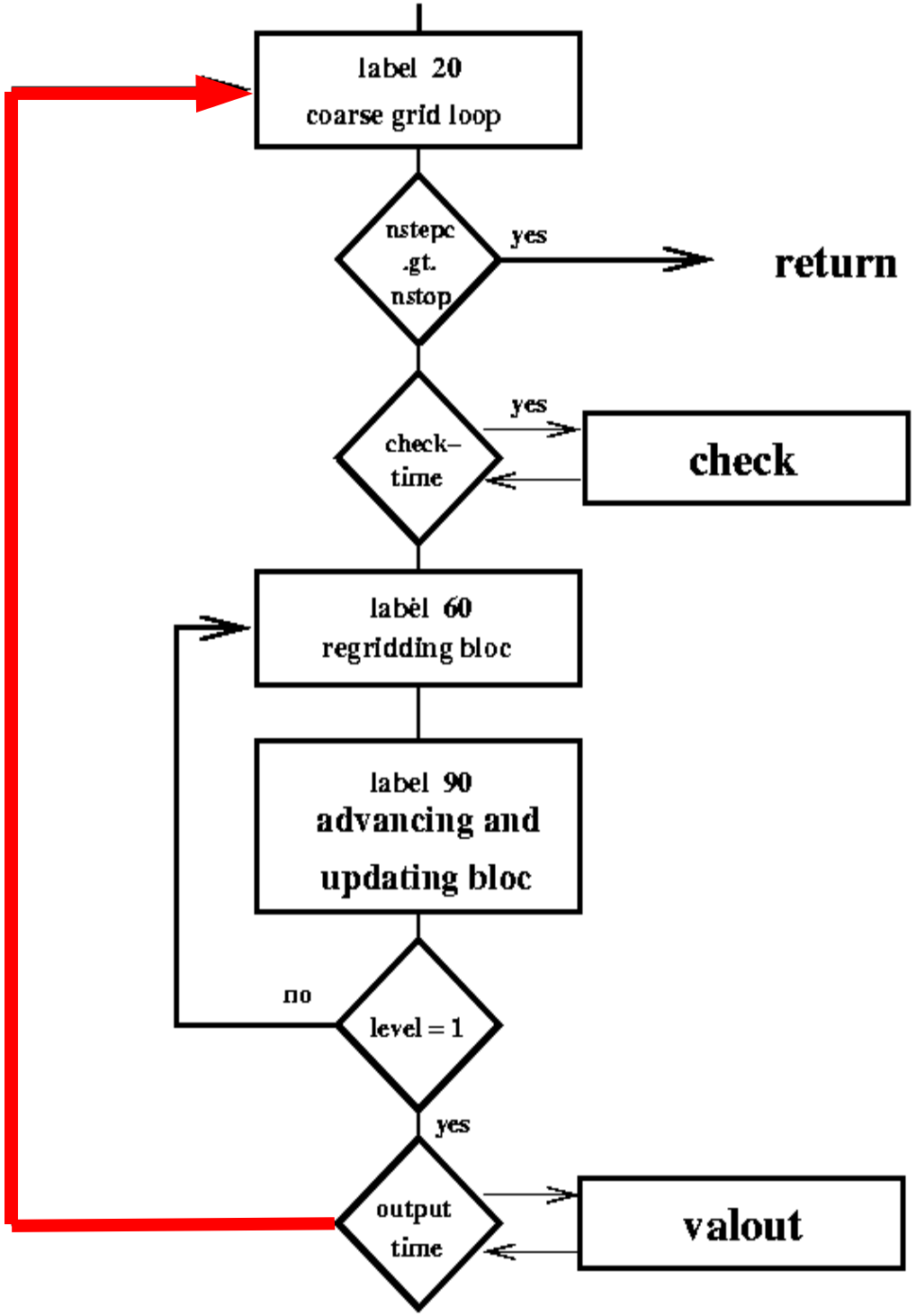
main driver routine

tick

organizes:

- integration (advancing block)
- remeshing (regridding block)
- consistence and boundaries between the meshes (updating block)
- data-output
- checking

coarse grid integration cycle
nstepc = nstepc + 1



subroutine **tick**
advancing and updating block

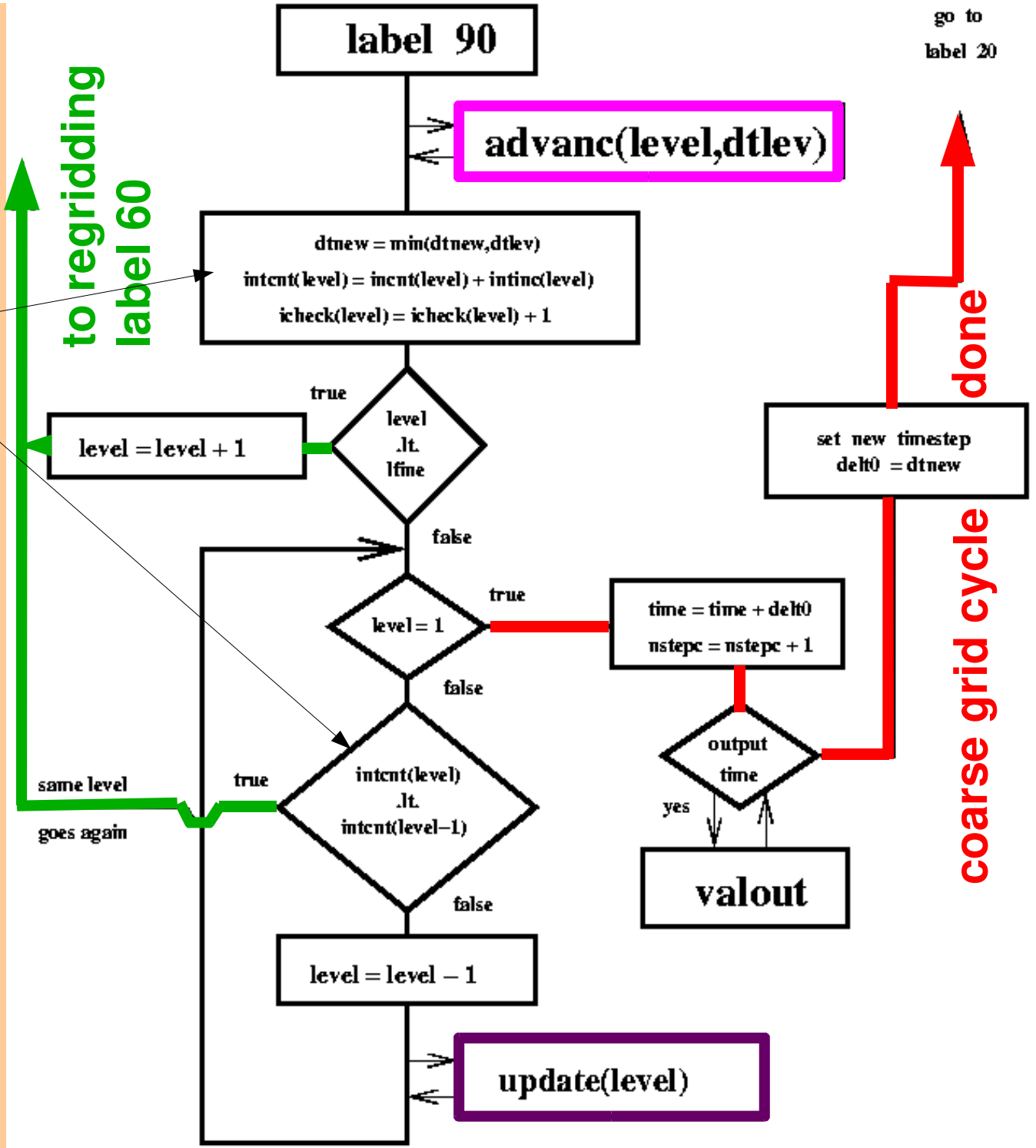
intctn(level):
integration counter

advanc(level,dtlev)

integrates all blocks at level level with timestep dtlev

update(level)

garantiesconsistence and treat boundaries between the meshes

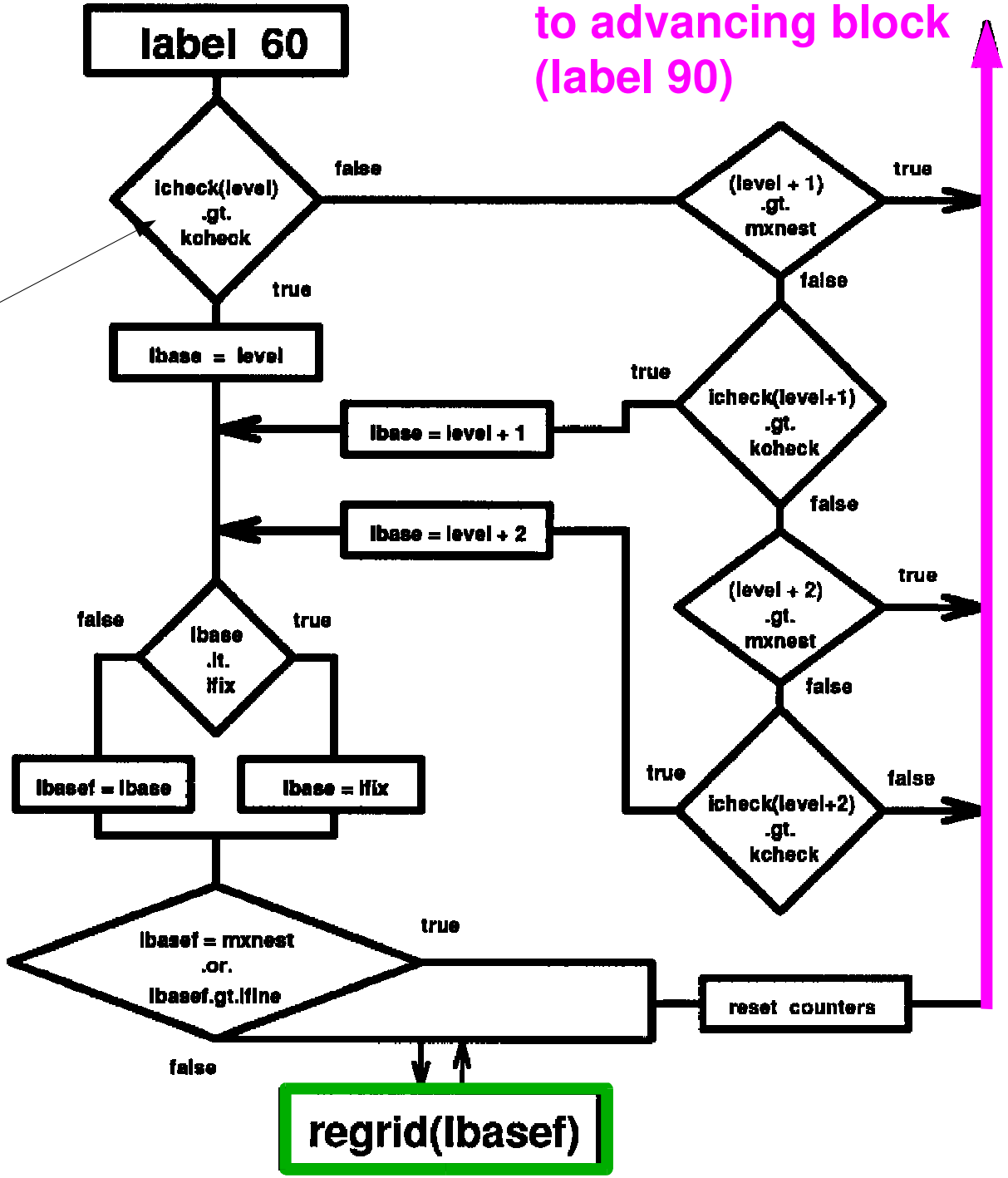


subroutine **tick**
regridding block

kcheck(level):
regridding counter

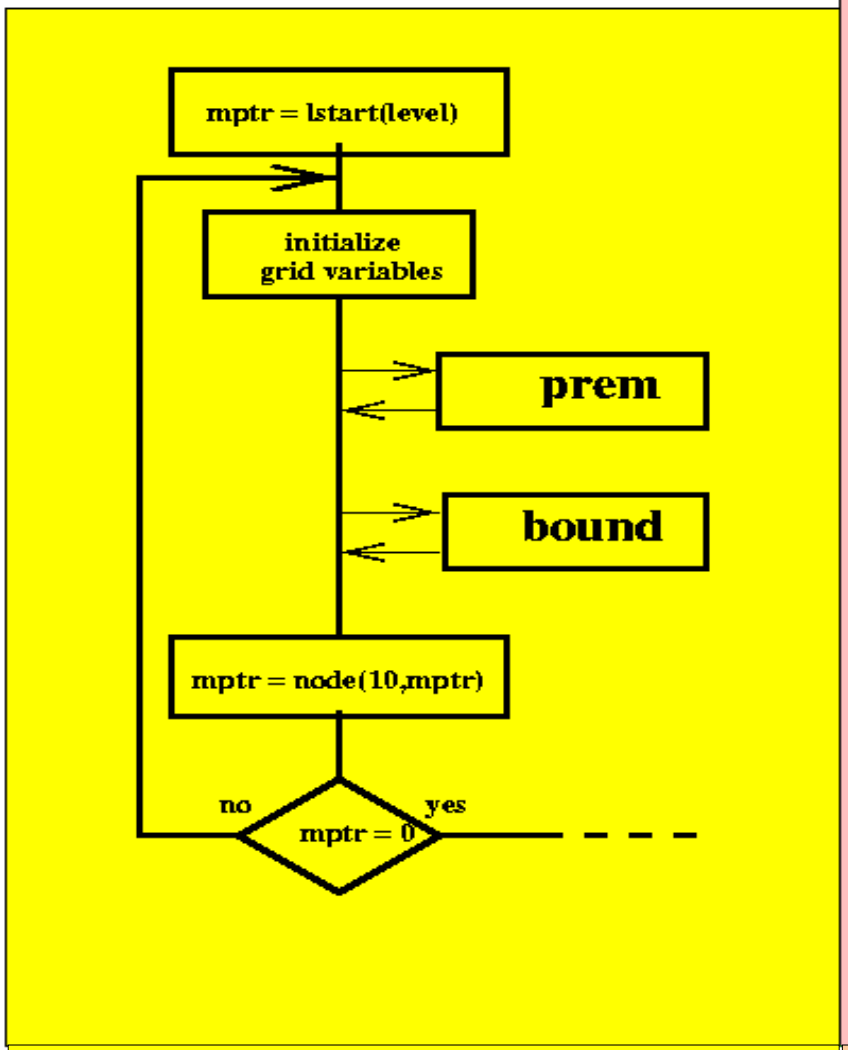
regrid (lbase)

driver for regridding process:
all levels higher than lbasef will be regridded

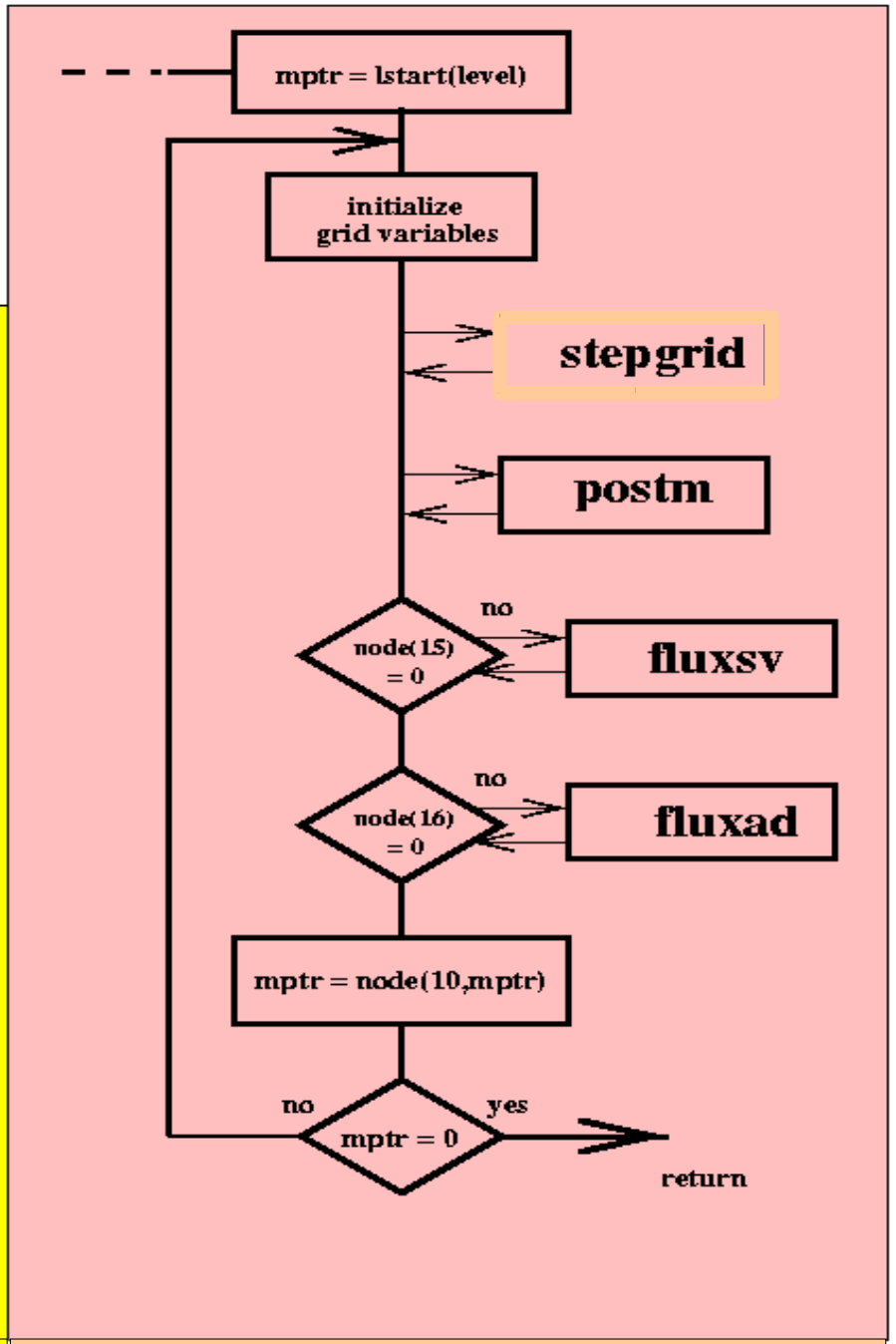


subroutine advanc(level)

integrates all blocks at refinement level 'level'



boundary loop: do all blocks



integration loop:do all blocks

integration

prepares conservation at internal boundaries

---> update

subroutine stepgrid (mptr)
integrates one block 'mptr'

calls in apropriate order the different integration routines:

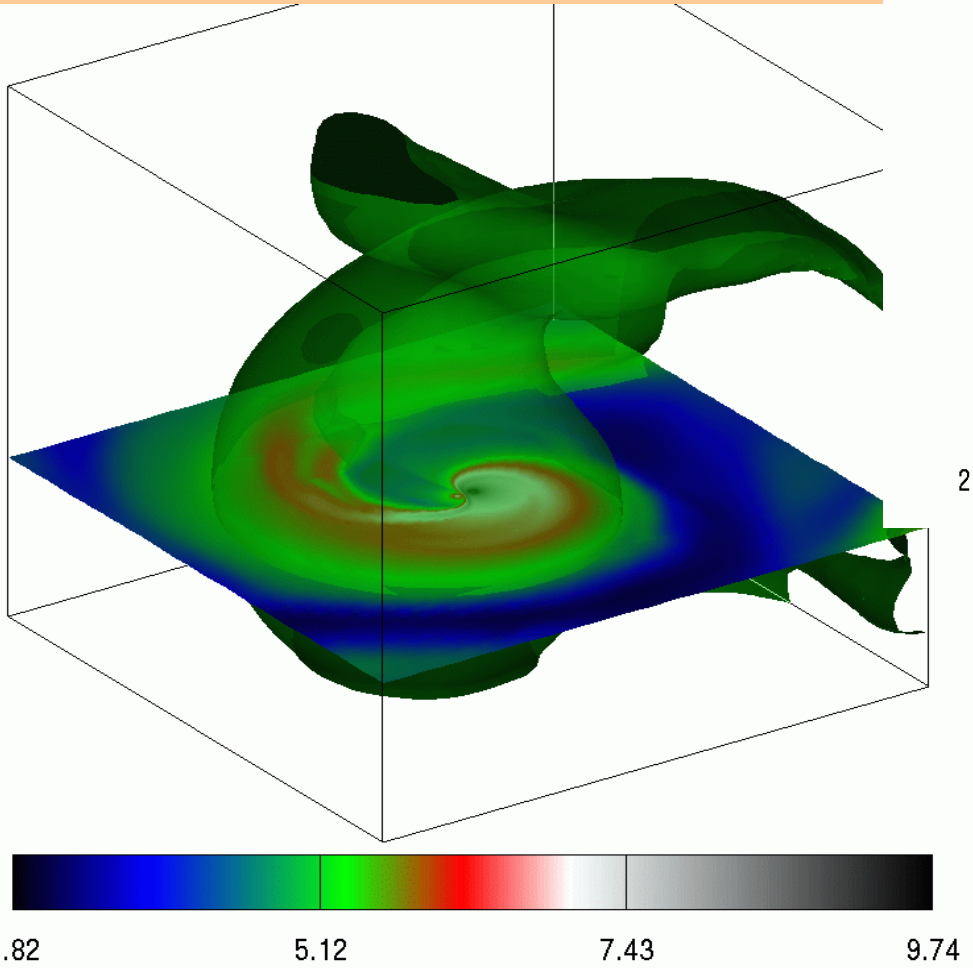
- cl_solv : solves the hyperbolic conservation law
- src_solv : solves for the source terms
- heat_cond_solv : solves for heat transer
- viscous_solv : solves for viscosity
- radtrans_solv : solves for radiative transfer
- ionstat_solv : solves for ionization states
- chemistry_solv : solves for chemistry

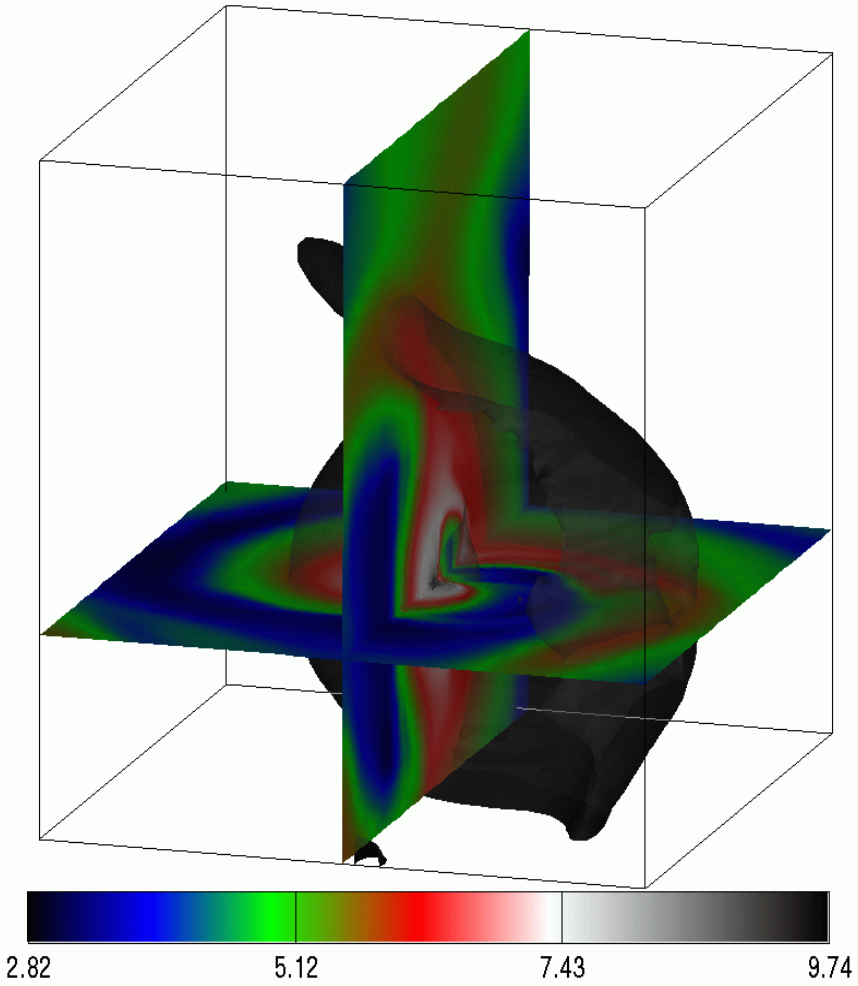
3D NEBEL

3D Radiation Transfer under Nebular Conditions

Synthetic Spectra and Ionization Structure

3D Density- and velocity- structure





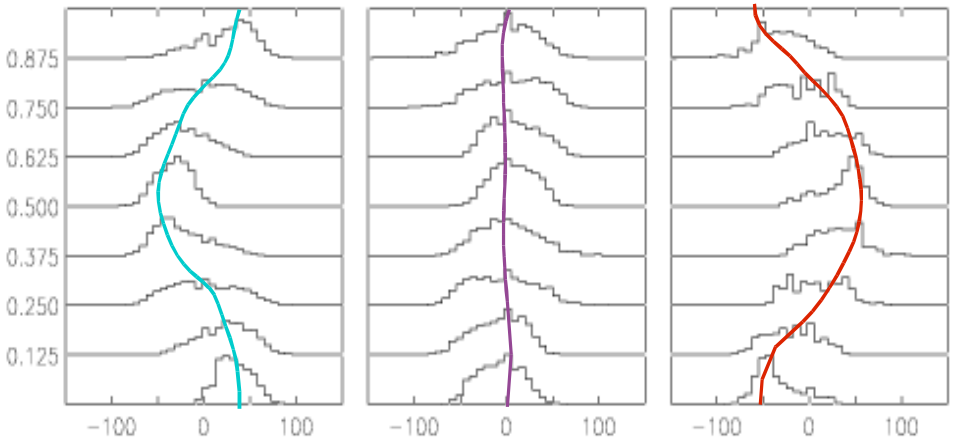


Figure 2. Line profile variation as computed for an observer in the orbital plane for the OIII line at 5008 Å for model *weak* with radiation sources of T = 90000 K and L = 100 $L_\odot$ (left), L = 1000 $L_\odot$ (middle), L = 10000 $L_\odot$ (right) for eight different phases (y-axis) and against relative velocity in km/s (x-axis). While for L = 100 $L_\odot$ the profile variation stems from the re-shaped circumstellar matter it merely reflects the undisturbed red giant wind for L = 10000 $L_\odot$.

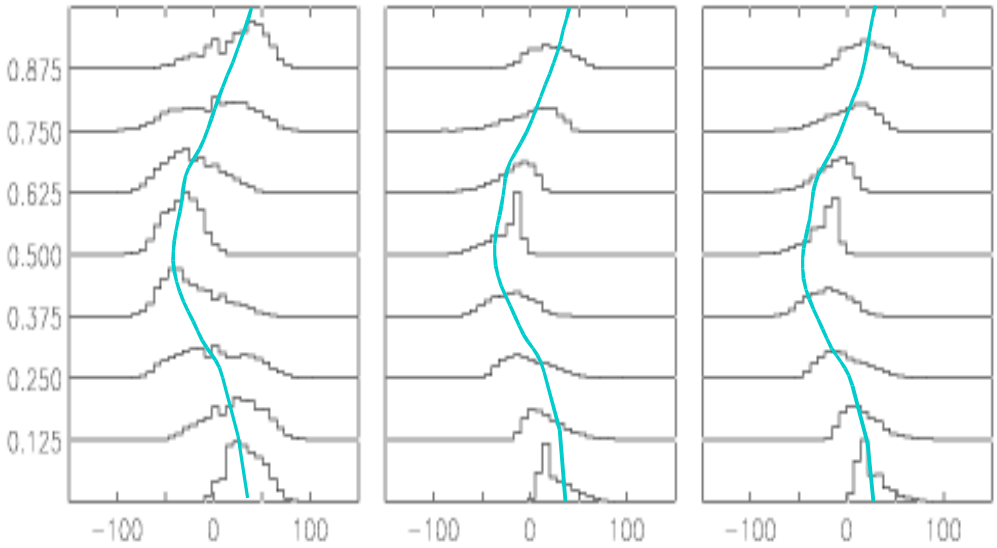


Figure 3. Probing the colliding winds interaction zone. The right ion makes it possible. From left to right: L=100 $L_\odot$, L=1000 $L_\odot$,L=10000 $L_\odot$ for model weak and T = 90000 K. The lines are 5008 OIII, 1242 NV, and 1031 OVI.

Taking the right ion, line profiles vary differently over orbit in model weak and strong, mirroring the different 3D density distribution:

- 3D distribution, model weak
- red giant wind only, model weak
- red giant + 3D distribution
- 3D distribution, model strong

weak $\phi_{min} \approx 0.5$ strong $\phi_{min} \approx 0.25$

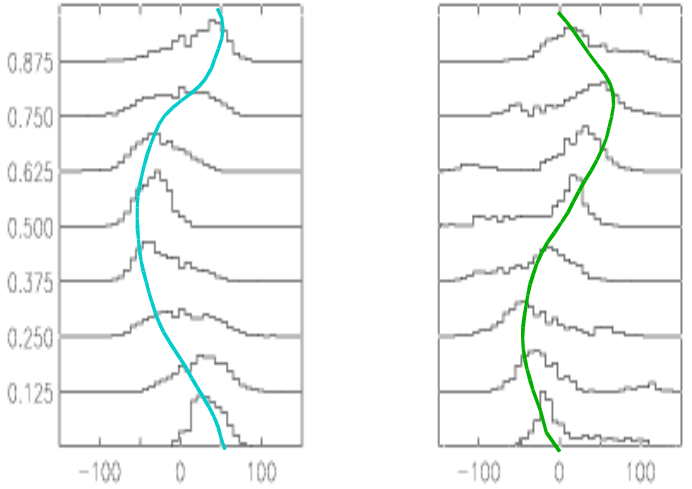
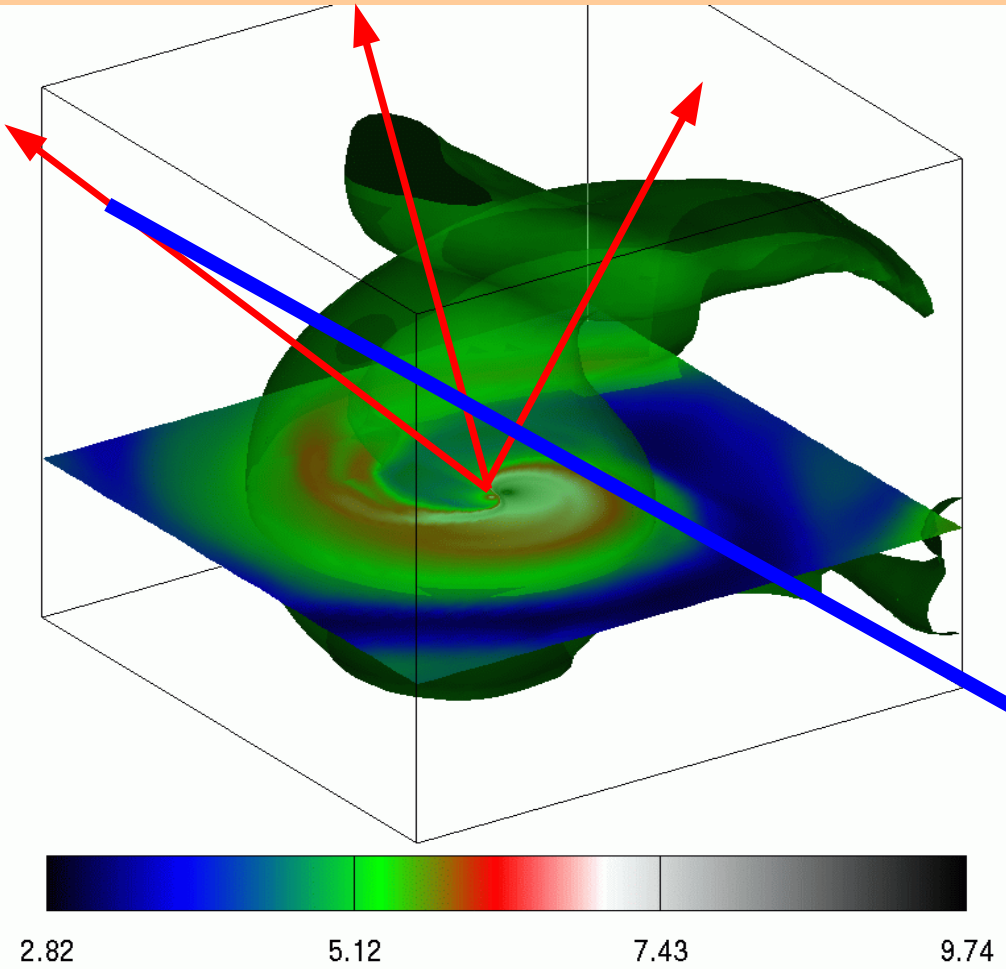


Figure 4. Considering the right ion, the phase at which maximum blue shift of the line profile is reached allows to distinguish between model strong and model weak. Shown is the OIII line at 5008 Å for model *weak* (left) and model *strong* (right) for a radiation source with L = 100 $L_\odot$ and T= 60000 K. The maximum blue shift for model *weak* occurs around phase 0.5, for model *strong* it occurs around phase 0.25.

3D Density- and velocity- structure



nebular conditions:
- no scattering
- time- independent
- all populations in the ground- level

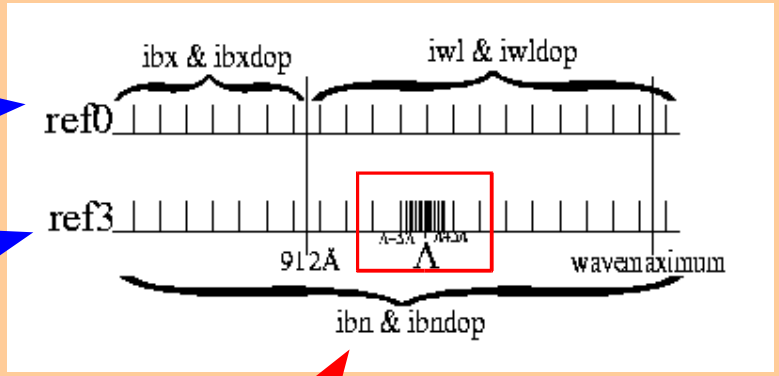
$$(\underline{n} \cdot \nabla) I^{\nu_f} = \eta^{\nu_f} - \kappa^{\nu_f} I^{\nu_f}$$

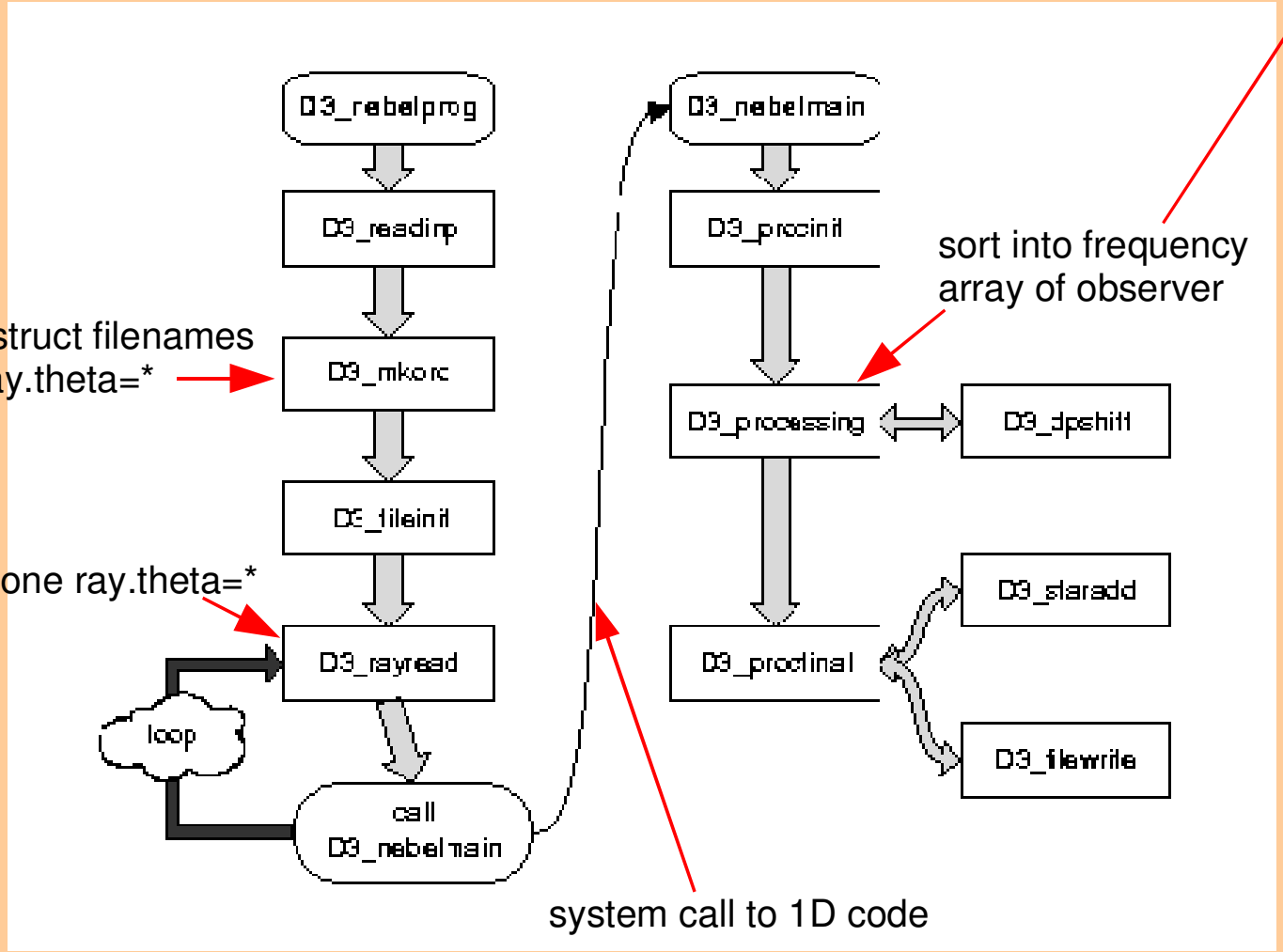
to observer

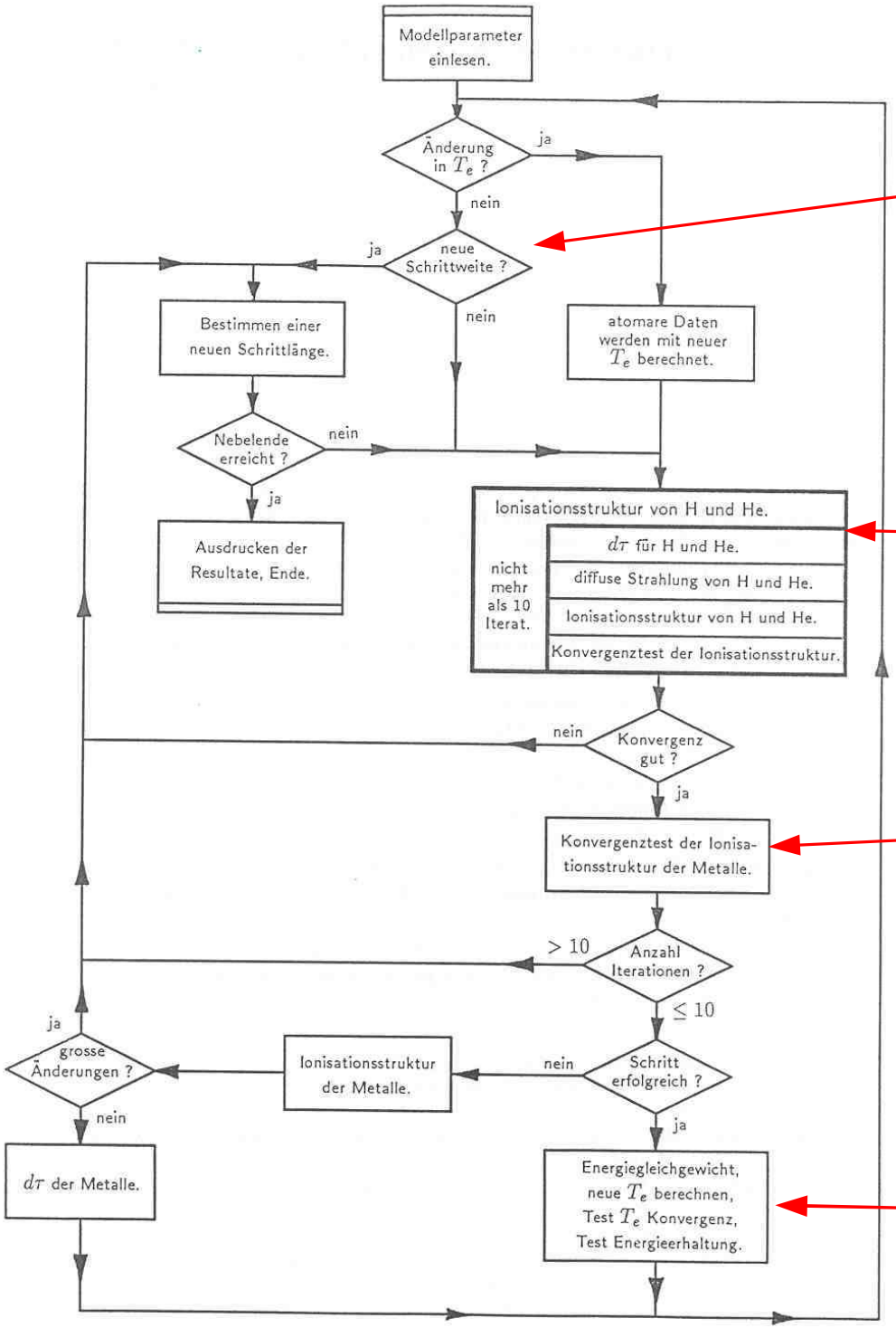
D3NEBEL schematically

frequency array for calculation along ray

frequency array for Doppler shifted spectrum, locally refined







adjust step size

ionization structure H,He

ionization structure metals

energy balance, T_e

TR3D

3D Optically thick Radiative Transfer

The radiative transfer equation in TR3D

$$\underline{n} \cdot \nabla_x I^\nu = \eta^\nu - \underbrace{(\kappa^\nu + \lambda^\nu)}_{\chi} I^\nu + \lambda^\nu \int_{S^2} d\Omega\, I^\nu$$

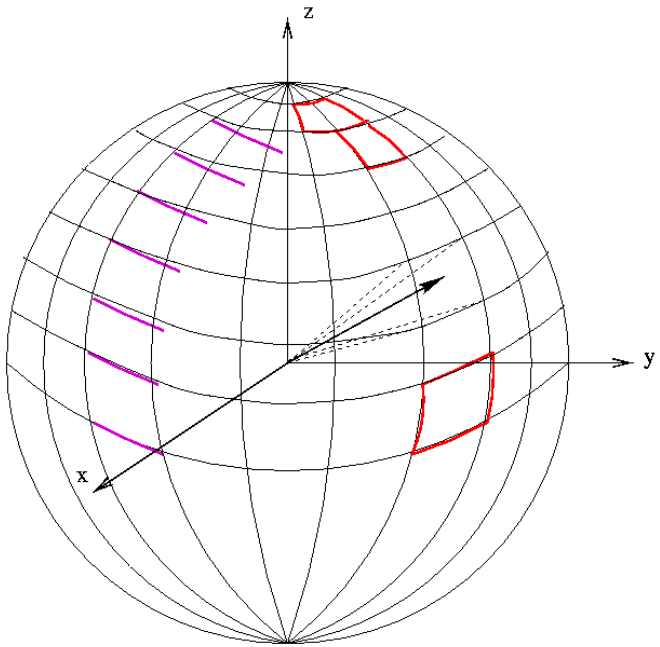
$$n \in S^2,\ I^\nu = I^\nu(\boldsymbol{x}, \boldsymbol{n})$$

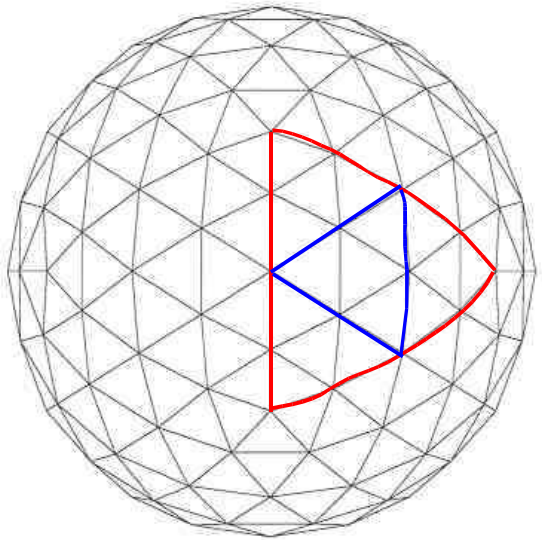
I^ν: specific (directed) intensity
η: emission coefficient
κ: absorption coefficient
λ: scattering coefficient

x: position in space
n: direction of propagation
S^2: unite sphere in 3D

Need discretization in ordinates

$$\lambda^{\nu} \int_{S^2} d\,\Omega\, I^{\nu}$$

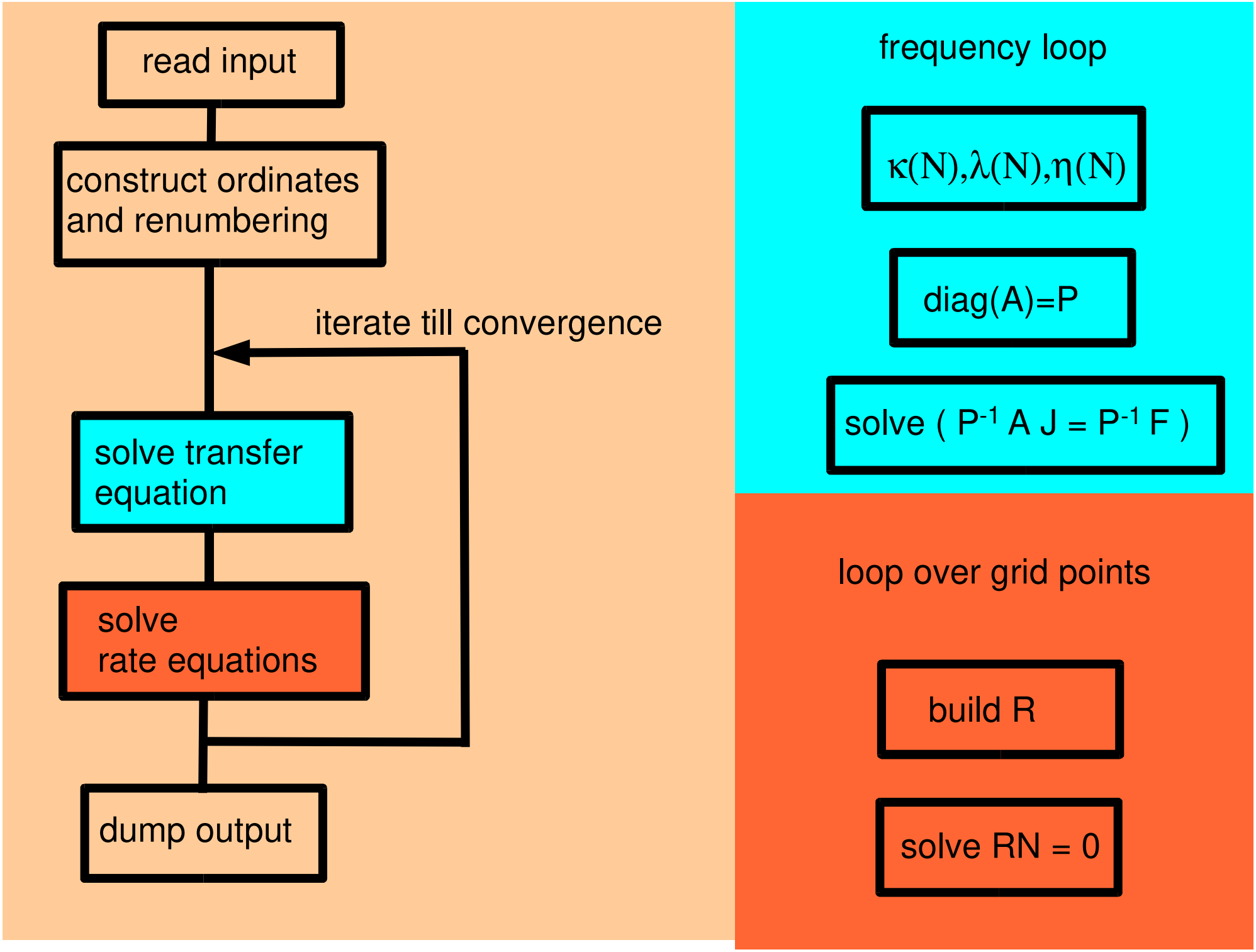




$$\underline{n} \cdot \nabla_x I^\nu = \eta^\nu - \underbrace{(\kappa^\nu + \lambda^\nu)}_{\chi} I^\nu + \lambda^\nu \int_{S^2} d\Omega\, I^\nu$$

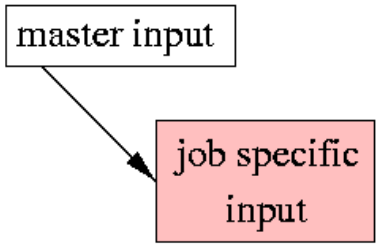
A linear system $A_h J_h = E_h$

At present, we solve $A_h J_h = F_h$ by a BiCGStab algorithm with diagonal preconditioning

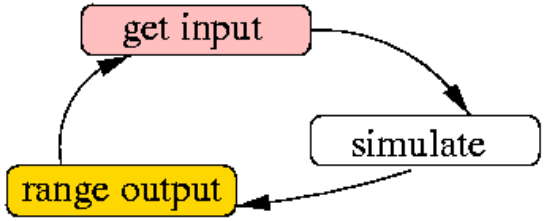

read input
construct ordinates and renumbering
iterate till convergence
solve transfer equation
solve rate equations
dump output
frequency loop
κ(N),λ(N),η(N)
diag(A)=P
solve (P-1 A J = P-1 F)
loop over grid points
build R
solve RN = 0

Data management and simulation control

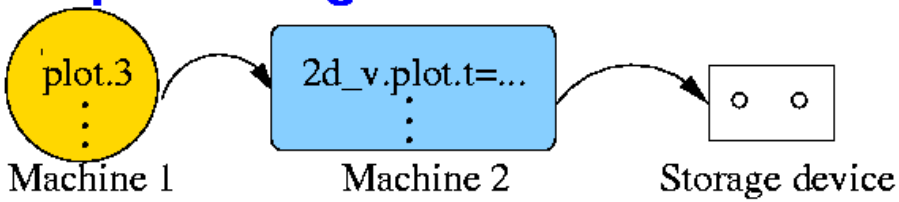
adapt input



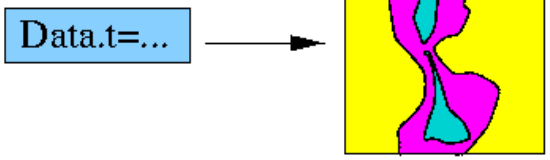
control simulation



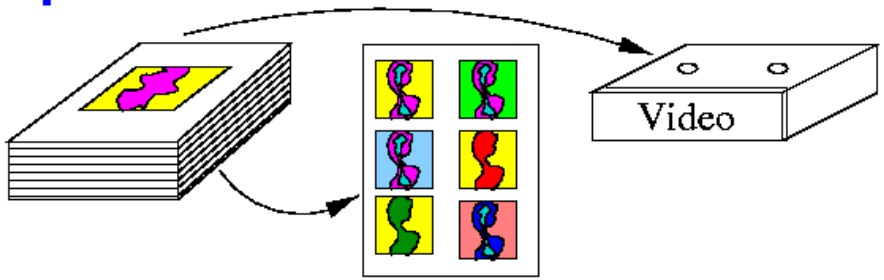
output management



graphics / statistics



compress information



A- MAZE

- provides multi- dimensional adaptive mesh codes for MHD and radiative transfer
- has proven to be able to simulate and visualize complex astrophysical objects
- is universal and easy to adapt to new problems
- is based on modern mathematical theory on numerical methods
- is built modeluar and provides an excellent basis to include algorithmes describing different physics
- enjoy your work with A-MAZE !