ASPECTS OF TURBULENCE IN ASTROPHYSICS

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Abstract

Turbulence in astrophysics is a key ingredient for the dynamical understanding of many objects. Birth, life, and death of stars, as only one example, crucially depend on it. We give a short overview of the different turbulent regimes encountered in different astrophysical objects. We describe some aspects of supersonic turbulence, i.e. turbulence with root mean square Mach number clearly above 1 and dissipation is mostly due to shocks. Finally, we discuss some issues related to numerical realization.

1 Introduction

Turbulence is ubiquitous in astrophysics. It has its peculiarities, however, bringing new, interesting facets into the field of research. The first peculiarity is its dynamical interaction with other physical processes under nearly all circumstances. Magnetic and radiative fields, gravity, nuclear, and chemical reactions substantially contribute to the momentum and energy balance. The second peculiarity is the eminent dynamical role turbulence plays for the evolution of many astrophysical objects. A third peculiarity results from the fact that theoretical or numerical results are hard to confront with laboratory measurements due to the extreme, nonterrestrial conditions under which the physical and chemical processes take place. Indeed, the essentially single source of information astrophysicists have on turbulence in outer space is the light from the object, either as a picture or merely as a spectrum. Abundant as such data are, they contain only indirect information (like the width of spectral lines) integrated along the entire line of sight. Consequently, the analysis of this data with regard to turbulence is a research field in its own right (1,2).

2 Turbulence in astrophysics: an overview

A nice overview of different aspects of turbulence in astrophysics provide the conference proceedings (3,4,5).

2.1 Stellar structure and evolution

To understand the chemical evolution of the universe stellar evolution must be understood, as essentially all chemical elements beyond Beryllium are produced by nuclear reactions in stars (6). Essential for the understanding of stellar evolution is the transport of energy and angular momentum within stars. Energy produced by nuclear fusion is transported outwards in essentially two ways: convection and radiation. Angular momentum is also transported by turbulent convection, losses (by stellar winds) or gains (by accretion from a binary companion) at the stellar surface thus affect the rotation profile throughout the star. Magnetic fields can substantially enhance the efficiency of the transport. - Turbulence also plays a crucial role for the production of magnetic fields in a stellar dynamo (5). Turbulence also transports newly produced chemical elements from the central region of the star to its outer layers (7). The associated change in chemical composition of the outer layers can affect the mass loss and thus the further evolution of the star. The mutual interactions of the different processes, which happen on different scales and in different locations within the star, have not yet emerged into a unique, complete picture. A lot of progress has, however, been achieved in the last few years. The far reaching consequences sketched at the beginning of the paragraph make the proper treatment of turbulence, turbulent mixing, and the interaction of the turbulence with the radiation- and/or magnetic field of the star a key topic in astrophysics.

2.2 Star formation

Star formation today takes place only in molecular clouds: cold (10K), high density (10^2-10^4 part./cm^3) regions of interstellar space (2–20 pc, 1 pc = $3*10^{18}$ cm) that are molecular in nature and contain 10^2 - 10^4 solar masses. From observations it is further known that these clouds are turbulent. Compressible turbulence is thought to regulate the efficiency at which stars form in such clouds. Turbulent pressure prevents global collapse of the cloud, which would result in a much higher star formation efficiency than what is observed. Locally, turbulence leads to density enhancements from which individual stars or small groups of stars can form by cooling and subsequent gravitational collapse. Whether the mass distribution of the newly formed stars (the initial mass function or IMF) is directly set by the turbulence is not clear. The question is of fundamental importance as the initial mass of a star is crucial for its evolution (e.g. life time and synthesis of chemical elements). Another important question is the driving of the turbulence, the energy source and injection scale. For reviews and further references we refer to (8-10) .

2.3 Accretion, winds, and jets

Accretion of mass and angular momentum onto a star or a black hole is one of the crucial processes in astrophysics. Turbulence plays a decisive role in this process. Conservation of angular momentum hinders direct infall of matter onto accreting objects. Accretion-disks form in which the fluid parcels have nearly Keplerian orbits. The orbital velocity is highly supersonic (up to Mach 100). Mass is brought inwards by the transport of angular momentum outwards. One mechanism for this is viscous dissipation provided by turbulence. Commonly it is believed that magnetic shear provides the mechanism that excites and drives the turbulence (11-13). Strong magnetic fields can be generated in an effective dynamo (5) at the same time. Spiral shocks and disk winds provide an additional source of angular momentum transport.

Outflows or mass ejections are not only essential for stellar evolution but also for the chemical enrichement of the universe. Moreover, they are principally linked to high-energy events such as supernova, -ray-bursts and quasars. Often, they are driven by external forces: momentum transfer from photons by scattering in spectral lines or on dust grains. Another important mechanism is centrifugal acceleration along magnetic field lines which fixes the matter to rigid rotation (e.g. winds and jets from accretion disks). Turbulence in outflows presumably originates from unstable driving (14). Most likely this process leads to a highly fragmented medium, high density knots, some kind of supersonic turbulence described in section 3.

2.4 Supernovae

We distinguish between two different classes of supernovae which both provide 10^{51} ergs explosion energy and 10^{49} ergs in light emission. In both classes, turbulence seems to play a principal role.

The first class is a thermonuclear runaway (type Ia in the phenomenological classification), which burns within less than a second about 1.4 solar masses of oxygen and carbon to iron. The turbulent reactive burning front probably develops from a combustion to a detonation regime (see e.g. Röpke and Hillebrandt in (4)).

The second class results from the gravitational collapse of the iron core of a massive star at the very end of its evolution. Most of the released gravitational energy is stored in neutrini. Only about 1 per mille of the energy is initially in the flow. Within less than a second after collapse and subsequent stabilization of the core in a neutron star, the neutrini start to dump energy into the gas, leading to strong convection. The still falling rest of the star gets shocked. As pointed out by (15) and (16), this can trigger a rapidly growing vortical-acoustic cycle: vorticity is generated at the kinked accretion shock, transported into the core, where it is scattered. The scattering produces strong sound waves which kink the shock even more when interacting with it. Moreover, the sound-waves strongly influence the energy dumping into the gas since the heating/cooling by the neutrini is proportional to T^6 (T: Temperature). All these processes combine to a so far very poorly understood gravo-neutrino-magnetohydrodynamical turbulence which eventually triggers the explosion of the entire star.

3 Aspects of supersonic Turbulence

We use the term 'supersonic turbulence' for turbulence with root mean square Mach number larger than one, $M_{rms} > 1$. Turbulence in this regime clearly deviates from the incompressible picture and is also much less understood. Most progress has been made for isothermal conditions (γ=1) on which we focus in the following. The results we are going to summarize are based on numerical simulations for the most part. Isothermal supersonic turbulence is thought to be a good first order model for the interstellar medium (ISM, Section 2.3) and, partly, stellar winds and jets (Section 2.4). An important aspect of the driving of this turbulence is the collision of two flows (e.g. faster and slower parts of radiatively driven stellar winds, wind-wind, supernova ejecta-ISM). We discuss a very simple model of such collision zones in Section 3.2.

3.1 Isothermal supersonic turbulence

Isothermal supersonic turbulence is characterised by the presence of shocks, strong density and velocity contrasts, and a patchy appearance of the flow (figure 1). Density has a log-normal distribution (figure 2; here γ=1 is crucial, (17,18)), correlation with velocity is essentially zero. Shocks account for at least 70% of the total energy loss (19,20). In driven turbulence, the strongest shocks dissipate most of the energy, while in decaying turbulence energy is dissipated by a large number of weak shocks. In the later case, the kinetic energy decays as t^{-h} ,with $0.85 < h < 1.2$ (21). The patchy spatial pattern formed by the shocks scales with the energy injection scale (8). For the velocity structure functions of driven isothermal turbulence, a unifying description from subsonic to supersonic has been proposed (22,23). Only one parameter, the Hausdorff dimension D of the most dissipative structure, has to be adapted as a function of M_{rms}, from D=1 (filaments) in the subsonic case to D=2 (sheets) in the supersonic case (M_{rms} = 10). Under non-isothermal conditions, values up to D=2.3 have been reported (24).

3.2 An example: the 2D slab

Whereas most studies of supersonic turbulence so far are based on computations in a periodic cubic box with artificial driving, we report now on a first systematic investigation of supersonic isothermal turbulence driven by colliding hypersonic flows (25). Such a scenario is both realistic and relevant. Stellar winds and stellar debris from exploded stars are among the most significant sources of energy for the ISM. Preliminary results of turbulence in colliding hypersonic flows which include strong (radiative) cooling can be found in (26-29).

Figure 1 shows the turbulent interaction zones from several simulations. Such zones are shown to be linearly unstable (30). In each simulation, the flows are anti-parallel but otherwise identical. The solution only depends on one single parameter, the upstream Mach number M_u, ranging from 5.5 to 87. The interaction zone is supersonically turbulent with characteristics as discussed in the last paragraph: $1 < M_{rms} < 16$, log-normal density distribution (figure 2), patchy appearance.

Assuming self-similarity in infinite space and no correlation between density and velocity, (25) performed a dimensional analysis within the frame of isothermal Euler equations. The analysis reveals the mean density in the turbulent interaction zone to be independent of M_u while M_{rms} scales linearly with M_u. Numerically - including viscous dissipation implicitly (see section 4) - this result is confirmed and $<\rho> \sim 30\rho_u$, $M_{rms} \sim 0.2\, M_{up}$. This is in sharp contrast to the situation in 1D, where it can be shown that $<\rho> \sim \rho_u M_u^2$ and $M_{rms} = 0$.

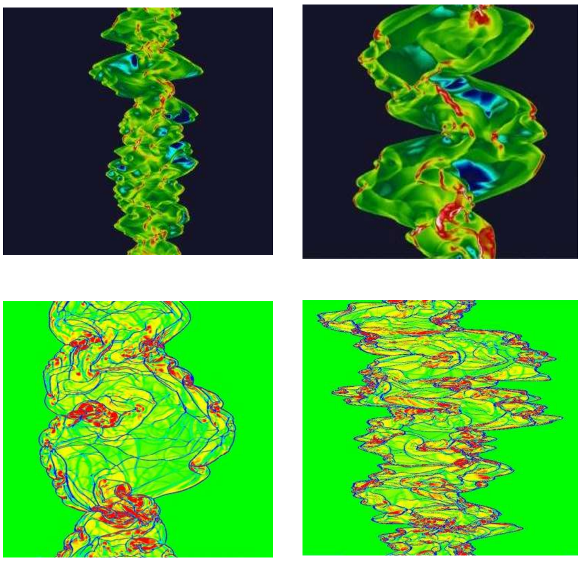

Figure 1: Supersonically turbulent interaction zone of head on colliding flows. Upper row: density for the case of $M_u = 20$ at an early time (left) and a late time (right). Lower row: velocity divergence for $M_u = 10$ flows (left and for $M_u = 30$ (right). The size of the patches and the wiggeling of the bounding shocks are closely linked and depend on M_{rms} and the width of the slab.

The analysis also brings a deeper insight into the nature of the turbulence. On dimensional grounds, the dissipated energy per unit volume and time must be proportional to $\rho_{diss} v_{diss}^3 l_{diss}^{-1}$. For the average column integrated value (average width of the slab: l_{cdl}) this results in $\varepsilon_{diss} \sim \rho_{diss} v_{diss}^3 l_{diss}^{-1} l_{cdl}$. With no other spatial scale and velocity scale present, it is then naturally to assume $l_{diss} \sim l_{slab}$, and $v_{diss} \sim v_{rms}$, leading to $\varepsilon_{diss} \sim \rho_{diss} v_{rms}^3$, or $\varepsilon_{diss} \sim \rho_{diss} M_u^3$. A closer analysis given in (18) reveals a very weak Mach-number dependence of ρ_{diss}.

The numerical results coincide with this analysis (figures 1, 3). The dissipation scale l_{diss} and the auto-correlation length of the shock wiggeling l_{corr} nicely scale linearly among each other and with the averaged width of the slab, l_{cdl}. All lengths scale with $M_u^{-0.6}$ (figure 3). This also confirms the eye-catching distribution of the patches and the shock-wiggeling which can be taken form figure 1. The patches and the shock-wiggles are big for large slabs and low Mach-numbers and small for small slabs and high Mach-numbers. The dissipation scale in this supersonically turbulent regime is clearly related to the mean distance between shocks and not to a viscous dissipation length. The good coincidence of the dimensional analysis based on the Euler equations with the numerical results which include implicitly viscous dissipation indeed proves the 'Euler-like' character of the turbulence in this regime.

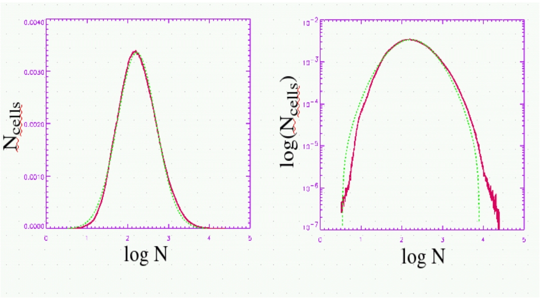

Figure 2: Log-normal density distribution of turbulent flow field shown in Figure 1. The slight deviation indicates probably a slight decay of the turbulence.

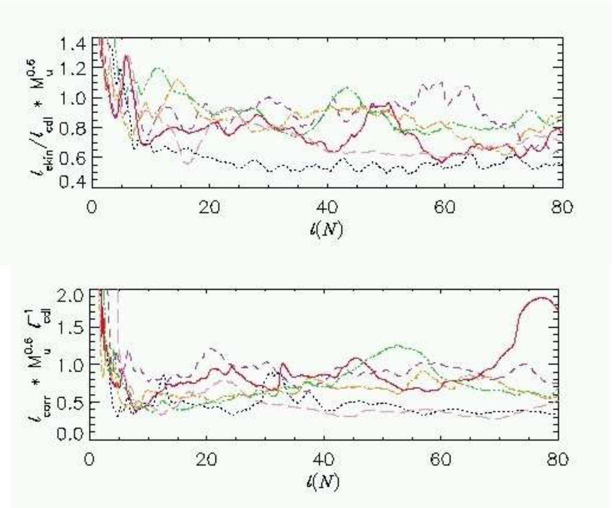

Figure 3: The length scale characterizing the dissipation, l_{ekin} (l_{diss} in the text) and the spatial scale of the wiggeling of the bounding shocks (l_{corr}) scale linearly with the width of the slab (l_{cdl}) and $M_u^{-0.6}$. Shown are the scalings against the mass accumulated in the slab (l(N)) for all runs of our sample. Inprints of the initial conditions die out at approximately l(N)~15, the finite size of the computational box start to affect the results at approximately l(N)=70.

4 Aspects of the numerical realization

In astrophysical hydrodynamical simulations, both high resolution finite volume shock capturing schemes (HRFV) such as PPM and statistical methods (so called 'smoothed particle hydrodynamics', SPH, see e.g. (8) for a review) are widely used. Other processes like gravity, radiation, and magnetic fields are mostly included using fractional step methods. Validating self-gravitating magneto-radiation-hydrodynamical simulations is difficult and most of the codes so far are not validated in a scientifically proper way.

In astrophysical simulations, Adaptive Mesh Refinement (AMR) has proven to be very useful. AMR helps, for example, to resolve turbulent burning layers in stars, which are - although geometrically thin - often the source of a convective overturn of large parts of the star. A turbulent molecular cloud starts to collapse due to gravity and cooling at an arbitrary moment of time at an arbitrary location in space, and one would like to follow that collapse over 3-4 orders of magnitude in space in a computationally well resolved manner. However, in connection with turbulence, AMR may also be a nuisance.

Compared to engineering, dynamical subgrid scale models (SGS) are less widely used in astrophysics. The reason for this lies mostly in the presence of shocks and/or other physical processes for which no proper SGS has been developed so far. Another reason is the notorious difficulty to combine SGS with AMR. Only recently, some attempts are made to use flamelets (e.g. Ewald and Peter in (4)) in simulating reactive burning fronts in supernovae and to combine dynamical SGS models with AMR (see e.g. Niemeyer, Schmit and Klingenberg in (4), (31)).

Computations of supersonic turbulence mostly use the MILES approach (Monotone Integrated Large Eddy Simulation). As pointed out in (32) and (33), HRFV-schemes act implicitly as SGS-models. In particular, the turbulent energy is passed correctly and without energy-jam from large scale to small scales, thus correctly mimicking the turbulent cascade. A larger study (34) confirms this for a large variety of schemes, such as MUSCL, (W)ENO, and Jameson. It also shows, however, the clear limitation of this approach. Many small scale flow features are suppressed by the large diffusivity of these schemes. The further use of an explicit dynamical SGS-model even makes the situation more worse. Having no other tools at hand, all the simulations discussed in Section 3 nevertheless used this approach. However, none of the only very few grid studies have shown convergence. Figure 4 shows the ratio of M_{rms} as computed on a fine and coarse grid for set of simulations used in section 3.2. Although 1280 and 2560 cells are used, the two M_{rms} differ by about 15 percent. Remarkably, the finer discretization shows more dissipation. In our view, this results from the better resolution of shocks, the main source of dissipation, on a finer grid.

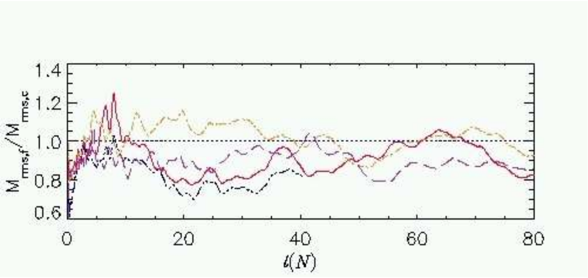

Figure 4: Difference in M_{rms} as computed on a fine and a coarse grid for our set of runs.

Clearly, the situation demands remedy. The authors make currently an attempt to develop appropriate methods which hopefully help to overcome the sketched problems in computation and validation.

5 Conclusions

Observations and theoretical arguments strongly support the view that turbulence is a key-process in astrophysics. The properties and evolution of a wide variety of astrophysical objects can only be understood if turbulence is taken properly into account. 'Properly' in astrophysics signifies that the turbulence is mostly compressible, even supersonic, and usually interwoven with other physical processes. Attempts to model these facets of turbulence have started only recently, in the wake of the steadily increasing computer power. More appropriate numerical tools still have to be developed. They must be able to cope with both, the chaotic dynamics of shocks and the subsonic turbulent cascade. Moreover, they must properly take into account the effect of external fields (radiation, magnetic, gravity) on the momentum and energy balance. Astrophysics is an excellent lab for studying many facets of turbulence and we predict a 'golden age' of turbulence research in astrophysics. To go along this path, we strongly encourage the interchange of models and results with other communities. We would like to thank ERCOFTAC to make this happening and possible.

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* A&A = Astronomy and Astrophysics
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